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Rating agencies, self-fulfilling prophecy and multiple equilibria?
An empirical model of the European sovereign debt crisis 2009-2011

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Abstract

We explore whether experiences during Europe's sovereign debt crisis support the notion that governments faced scenarios of self-fulfilling prophecy and multiple equilibria. To this end, we provide estimates of the effect of interest rates and other macroeconomic variables on sovereign debt ratings, and estimates of how ratings bear on interest rates. We detect a nonlinear effect of ratings on interest rates which is strong enough to generate multiple equilibria. The good equilibrium is stable, ratings are excellent and interest rates are low. A second unstable equilibrium marks a threshold beyond which the country falls into an insolvency trap from which it may only escape by exogenous intervention. Coefficient estimates suggest that countries should stay well within the A section of the rating scale in order to remain reasonably safe from being driven into eventual default.

Keywords

Eurozone, crisis, sovereign debt, credit spreads, bond yields, rating agencies, multiple equilibria, self-fulfilling prophecy.

JEL Classification

F3, G24, H6.
1 Introduction

Internet blogs are alive with conjectures of multiple equilibria and self-fulfilling prophecy as key characteristics of the European sovereign debt crisis, and with discussions of its implications. Academic journals feature an impressive list of refined models that may generate multiple equilibria. Interest in this topic existed well before the Great Recession, as the experience of 2007-2009 is generally referred to, but intensified while the twin crises gained traction. Policy discussions offer compelling advice as to what recipes could work in situations where good and bad equilibria coexist side by side. By contrast, little, if any, direct empirical evidence appears to have been put forward in support of the actual existence of multiple equilibria in Europe’s current turmoil.

This paper offers some evidence to this effect.

Section 2 surveys related work that explores multiple equilibria and self-fulfilling prophecy in the context of government debt. The next section describes a simple model in the spirit of Romer (2012), with roots in the seminal work of Calvo (1988), which provides a basis for our empirical analysis. This empirical analysis proceeds in two steps. We first draw on regression results advanced in previous work to shed light on whether self-fulfilling prophecy and multiple equilibria may be at work during the European sovereign debt crisis. In a second step we conduct our own, more detailed and direct analysis in which rating agencies and nonlinear relationships between interest rates and ratings play a key role. Section 6 presents some refinements of the empirical model and uses these to identify insolvency thresholds beyond which default appears unavoidable without outside help. The final section discusses

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3 See, for example Gerlach (2010) and De Grauwe (2011)

4 Two recent econometric papers bear indirectly on the issues of multiple equilibria and self-fulfilling prophecy in the current sovereign debt crisis. De Grauwe & Ji (2012b) find evidence "that a significant part of the surge in the spreads of the PIGS [...] countries in the Eurozone during 2010-11 was disconnected from underlying increases in the debt to GDP ratios, and was the result of negative market sentiments that became very strong since the end of 2010." This result is shown to apply to a wider set of fundamentals in De Grauwe & Ji (2012b). Similarly, von Hagen, Schuknecht & Wolswijk (2011) report that "markets penalise fiscal imbalances much more strongly after the Lehman default in September 2008 than before." Blanchard (2011) considers the 'facts' so convincing that he states: "post the 2008-09 crisis, the world economy is pregnant with multiple equilibria" as the first of the hard truths he learned from 2011.
caveats, sums up and concludes.

2 Related work

The field of debt crises due to multiple equilibria and self-fulfilling prophecies took off with the influential work of Calvo (1988). Even though models with multiple equilibria circulated much earlier, they were mostly considered to be theoretical artefacts of possibly misspecified models. Calvo kick-started the idea that multiple equilibria were a phenomenon worth analysing instead of discarding it as many others had done before him. His simple two-period model of pricing government debt highlights the fact that for specific parameter values multiple equilibria occur. Self-fulfilling expectations can lead to any one of them.

Alesina, Prati & Tabellini (1990) jump on that idea and empirically analyse the debt structure of Italy during the 1980s discussing the origins of and the remedies for self-fulfilling debt crises.

Cole & Kehoe (2000) extend Calvo’s model by embedding it into a Dynamic Stochastic General Equilibrium framework. They show that specific constellations of a country’s fundamental values such as the debt level, maturity structure and private capital stock can move it into a so-called crisis zone. Here, the probability of default is no longer a function of fundamental values but is determined by the beliefs of market participants. Cole & Kehoe also suggest certain policy actions for countries to cope with being in the crisis zone, such as reducing debt or increasing average maturity. However, they point out that the best strategy would be to never get into the crisis zone in the first place, i.e. to keep debt levels below a certain threshold. The authors also refer to their own, earlier work, Cole & Kehoe (1996), which provides empirical support for their model. Using a calibrated model they confirm that Mexico, in its 1994 crisis, was probably in such a crisis zone.

A similar argument is found in Masson (1999b), namely, that market sentiment and self-fulfilling expectations - not fundamental variables alone - explain the spread of the crisis in Mexico and East Asia in 1997. Masson (1999a) further develops the idea that crisis contagion can be more easily explained in an environment of multiple equilibria and self-fulfilling expectations.

Another early contribution in the area of sovereign debt crises comes from to Alesina, Broeck, Prati, Tabellini, Obstfeld & Rebelo (1992). Using

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5See also Romer (2012) who gives an excellent, brief introduction into the topic. A related field in the literature, worth mentioning, is that of self-fulfilling currency crisis, spearheaded by Krugman (1996).
regression analysis on a panel data set of 15 OECD countries they show that a selection of fundamental variables influences the perceived default probability. As a proxy for the latter variable the authors use either the ratio of the public interest rate to the private interest rate or the difference between those rates. The fundamentals chosen are the public debt ratio, the change in that ratio, industrial production and the average maturity of public debt. While their results do not give an entirely clear picture, and raise some questions,⁶ they emphasize the positive influence of the debt level on perceived default risk. They also argue that high debt levels lead to the possibility of multiple equilibria in the sense of a self-fulfilling confidence crisis.

A purely theoretical contribution on multiple equilibria in sovereign debt pricing is due to Detragiache (1996). His model emphasizes the necessity of a market that consists of many small investors in order to explain situations with multiple rational expectations equilibria. Pessimistic expectations of creditworthiness may then trigger a liquidity crisis.

Among the first to put rating agencies into the game, in the sense that ratings might have an influence on outcomes if multiple sunspot equilibria exist, were Kaminsky & Schmukler (2002). In a panel regression they show that sovereign debt ratings do not only affect the bond market but also spill over into the stock market. This effect is stronger during crises, which could be explained by the presence of multiple equilibria. As a consequence they claim that rating agencies contribute to the instability in emerging financial markets. Carlson & Hale (2005) argue that if rating agencies are present, multiple equilibria emerge in a market in which otherwise only one equilibrium would exist. The purely theoretical paper is an application of global game theory and features heterogeneous investors. Boot, Milbourn & Schmeits (2006) arrive at the opposite conclusion: ratings serve as a coordination mechanism in situations where multiple equilibria loom. Using a rational-herd argument, they show that if enough agents base their investment decisions on ratings, others rationally follow. Since ratings have economic consequences, they emphasize that the role of rating agencies is probably far greater than that of the self-proclaimed messenger.

After the outbreak of the recent financial and sovereign debt crises many new papers unsurprisingly appeared in this field. Besides our own work in Gärtner, Griesbach & Jung (2011), Arezki, Candelon & Sy (2011) also find a significant effect of sovereign rating news on credit markets during 2007-2010.

⁶There is a critical discussion appended to the paper with comments by Maurice Obstfeld, Sergio Rebelo, Martin Hellwig, Hans-Werner Sinn, etc. who, among other things, criticize the construction of the dependent variable, the risk premium.
This is in line with results provided by Kiff, Nowak & Schumacher (2012), who find that ratings affect the cost of funding sovereign issuers and are, therefore, a threat to stability in sovereign bond markets. Multiple equilibria and self-fulfilling prophecies are also addressed in De Grauwe (2011), De Grauwe & Ji (2012a), De Grauwe & Ji (2012b) and Corsetti & Dedola (2011).

3 A simple model of interest rate and sovereign bond ratings interaction

The backbone for our empirical analysis is provided by Romer’s (2012) structural adaption of Calvo (1988) optimizing model of sovereign debt crises. These models look at the interaction between interest rates on government bonds and expected probabilities of sovereign default, where, ceteris paribus, the interest rate is assumed to bear on the likelihood of default, and the likelihood of default affects the interest rate; the models analyse the rational-expectations equilibria that may arise in such a setting.

It should facilitate the interpretation of the empirical work to follow below if we take a brief look at the Romer model and its graphical representation. The model consists of two equilibrium conditions. The first one renders investors indifferent between some exogenous risk-free interest rate (or rate of return) \( i^* \) and the interest rate \( i \) for a government bond with an attached non-zero default probability of \( p \). If default is a one-off event, creditors are risk neutral, and the government only issues one-period bonds, this equilibrium condition reads \((1 - p)(1 + i) = 1 + i^*\), or

\[
p = \frac{i - i^*}{1 + i}
\] (1)

In a diagram that features the default probability on the horizontal axis and interest rates on the vertical axis, equation (1) is displayed as a hyperbola-shaped curve, which intersects the ordinate at the risk-free interest rate (see Figure 1).

The second equation focuses on the fact that, in each period, the government decides whether to service its debt or to go into default. This decision depends on the difference between its ability to pay, i.e. the tax revenue, and the required payment, i.e. \( i \times D \). For given tax revenue and debt \( D \), the higher the interest rate, the higher the probability of default. Thus

\[
p = F(i, D) \quad F_i, F_D > 0
\] (2)

Figure 1 displays this relationship as a Z-shaped line. Up to an interest rate \( i_1 \), servicing the debt is painless and there is no relevant risk of default.
Beyond $i_2$ servicing the debt would drain money from so many other critical policy areas that it would be political suicide. In this situation, going into default remains the only feasible option. In the interval between these interest rates the default risk increases monotonically.

When curves are positioned as in Figure 1, the model generates three equilibria. The first one is the point of intersection between the two lines in the lower left part of the diagram (point X). In this good equilibrium the interest rate is low and the government is likely to honour its commitments. The second equilibrium, at point Y, implies a substantial spread between the interest rate the government pays and the risk-free alternative, reflecting a significant risk of default. In the third equilibrium, interest rates rise so high that default becomes certain. So nobody purchases the government’s debt and the government is forced to default. This ‘equilibrium’ cannot be identified by a point since the market for this country’s debt titles has broken down and no interest rate is determined.

Equations (1) and (2) describe an equilibrium model but do not really say anything about dynamics. As spelled out by Romer (2012, pp. 637 ff.), however, under plausible assumptions such as permitting a lagged response by interest rates to changes in default probabilities, the first and the third equilibrium are stable. The one in the middle, point Y, however, is unstable and functions as a threshold. Once this threshold has been passed, default becomes very much inevitable.

Of course, three equilibria only obtain when the relative curve positions

\footnote{This would follow from discarding the notion of rational expectations and replacing it by some adaptive scheme.}
are as shown in Figure 1. If we move the $i$ curve far enough up, the good equilibrium disappears and, without outside intervention, default cannot be avoided. If we move the $i$ curve sufficiently far down, we are left with the good equilibrium only, which is then globally stable.

4 An empirical model of interest rate and sovereign bond ratings interaction: Take 1

We now ask whether the empirical data support an interpretation of recent developments in financial markets in line with Romer’s model. The model employed here differs from the Calvo-Romer scenario in two ways. First, we replace the default probability of sovereigns by sovereign bond ratings. This variable is easily observed and measured and it permits us to discuss how rating errors or abuse, an issue that has had a lot of attention in Europe in particular, may affect the dynamics of a sovereign debt crisis. Second, acknowledging the evidence of how expectations may be formed in financial markets during normal times, and acknowledging the often expressed view that markets in the aftermath of the financial crisis often appeared to be driven by panic and fear, or even schizophrenia, rather than rationality, we look beyond rational expectations equilibria to permit bandwagon and herd behaviour to allow for institutional influences, and thus warrant a richer set of dynamics.

Our model comprises the same two propositions as the Romer model. First, interest rates $i$ on government debt are affected by the expected probability of default as signalled by the sovereign bond rating $r$ plus, potentially, a vector of macroeconomic and political variables $M$ which market participants may assume to affect the risk-free interest rate and which would, thus, affect the position of the interest rate line in Figure 1. We also leave room for other functional forms, since the hyperbola derived as equation (1) only obtains under a set of restrictive assumptions:

$$i = \Omega(r, M) \quad \Omega_r > 0$$

Second, the probability of default as measured by sovereign bond ratings is affected by a vector of macroeconomic and political variables $N$, from which we single out the interest rate to be determined endogenously in the

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8See Haruvy, Lahav & Noussair (2007, p. 1901) who conclude: “We find that individuals’ beliefs about prices are adaptive”.

context of this simple model, and an error term $\epsilon$, permitting errors or biases to be included:

$$r = \Psi(i, N, \epsilon) \quad \Psi_i > 0$$

(4)

Rather than looking at the properties of this adaption of the Romer model in the abstract, we draw on the econometric estimates provided in Gärtnert, Griesbach & Jung (2011), henceforth referred to as GGJ, as some sort of stepping stone towards the indigenous empirical work presented in Section 5.

4.1 Estimates and graphical representation

The GGJ estimates derived from a panel of 262 observations for 26 OECD countries that may be interpreted as linearizations of equations (3) and (4) around their means. The estimate of equation (3) is

$$i - i_D = -8.26 + 0.40 r$$

(0.67) (0.03)

$R^2 = 0.34 \quad n = 262$

(5)

where $i - i_D$ denotes the credit spread of the respective country versus Germany. The vector $M$ of variables affecting the risk-free interest rate is redundant since the German interest rate is used directly as a representative of the risk-free rate, with its coefficient being constrained to 1. The equation

9Endogenizing other macroeconomic variables such as income, inflation, deficits or debt would require the integration of the debt crisis model into a full-scale macroeconomic model. Doing so might open a Pandora’s box of issues on which economists do not agree, ranging from the magnitude of multipliers to the relevance of Ricardian equivalence. In order to sidestep this, we stick with a conservative approach here, and settle for an informal discussion of how the endogenization of selected macroeconomic variables may affect our results. See section 6.

10In an effort to remain parsimonious, equation (4) refrains from listing other variables, such as income, deficits and inflation, that may affect a country’s ability to service and repay its debt. Changes in these variables affect the position of the rating curve.

11The results reported here are OLS regressions, though GGJ present robustness checks with other methods as well. For OLS not to produce biased and inconsistent estimates of a simultaneous equations system as given by (3) and (4), some endogenous variable on the right-hand side needs to be lagged and, thus, predetermined in order to make the system recursive. This is indeed proposed with regard to equation (4), where ratings are assumed to respond to changes in the interest rate with a lag. See equation (6).

12The linear conversion of the rating scale used in GGJ, running from D=1 to AAA=21 is the reverse of the scale used in this paper (see Appendix A.1 below). Constant terms and signs of the GGJ regressions reported here have been adjusted to make results comparable.
suggests that each downgrade of a country’s credit standing by one notch raises the interest on a country’s public debt by 40 basis points.

\[
    r = -2.40 + 0.46 i_{-1} + bN
\]

\[
    (0.77) \quad (0.08)
\]

\[
    R^2 = 0.67 \quad n = 262
\]

(6)

where \(b\) is a vector of coefficient estimates not shown here. According to these estimates, rating agencies tend to downgrade a country’s credit standing by almost half a notch if the interest rate for government bonds increases by one percentage point.

4.2 Stability and self-fulfilling prophecy?

Figure 2 visualizes the regression lines along with confidence bands in an \(r\)-\(i\) diagram. The \(i\) line shows how interest rates respond to sovereign bond ratings. The \(r\) line shows how changes in the credit costs for governments affect sovereign bond ratings. The point of intersection – or the area of intersection, if we include the confidence bands – marks the unique rational ratings equilibrium. Only here does a sovereign rating generate the very interest rate that, in turn, justifies this rating.

![Figure 2: Estimates of ratings and interest rate equations from Gärtner, Griesbach & Jung (2011), Tables 2 and 3. Thin lines show 95% confidence bands.](image)

Suspicious have been put forward that sovereign bond ratings may not be all that rational. They may, deliberately or not, contain systematic biases or may be misused for other purposes. In this case questions of stability

10
may arise that are excluded by assumption in a rational ratings (i.e. rational expectations) scenario. We may distinguish two cases:

- The credit rating agency commits an error of judgement; this would be captured by the white noise error term in the estimation equation. So, without relying on appropriate deteriorations of fundamental variables, it issues an unsolicited downgrade from the equilibrium value \( r_x \) to \( r_1 > r_x \). Investors respond to this signal of an increased default probability by requiring a higher interest rate \( i_1 > i_x \) (Figure 3). This actually raises the risk of default, but not by as much as the initial, erroneously given rating suggested. So while the initial rating hike triggers a self-fulfilling response, it is quantitatively inadequate. Since the error is temporary, by assumption, the rating responds to the new interest rate \( i_1 \) by reverting via \( r_2 \) towards its initial value \( r_x \). Evidently the equilibrium derived from the estimated coefficients is stable since they imply an \( i \) curve that is flatter than the \( r \) curve.\(^{13}\)

- The rating agency attempts to manipulate the market by deviating permanently from the previously applied and correct rating algorithm. This might be captured by a change in the estimation equation’s constant term. An unjustified downgrade shifts the \( r \) curve to the right into the dashed position and the equilibrium moves up into point 1 (Figure 4). This equilibrium is stable as well and will be approached monotonously from below. So while the downgrade was, by assumption, arbitrary and not justified by fundamentals, it is self-fulfilling in the sense that both the rating and the interest rate start to rise and remain permanently higher.\(^{14}\)

In a transparent, well-informed market this ‘equilibrium’ would not persist, of course. The initial solid \( r \) curve reflects the true or correct relationship between a country’s likelihood of default and the interest rate. This relationship still applies, since we assumed the shift into the dashed position to be arbitrary. But then, given the initial set of fundamentals, the new equilibrium rating \( r_1 \) would only be justified at an interest rate \( i'_1 \), which is much higher than \( i_1 \), the rate that actually obtains. Investors would detect a gap between the risk assessment of the rating agency and the true risk, deduct some discount from the credit rating, and the interest rate would return to

\(^{13}\)Note that the \( r \) curve given by equation 6 reads as \( i_{-1} = 5.22 + 2.17r - \frac{b}{0.45N} \) when solved for \( i_{-1} \).

\(^{14}\)Note that with the numerical transformation used here, downgrades imply rising rating values.
Figure 3: Stability of empirical rating and spread relationships.

Figure 4: Rating manipulations with different $i$ lines.
In the course of the eurozone’s sovereign debt crisis a steadily increasing number of observers started to doubt whether the level of transparency and information required by this line of argument matches reality in this context. Factors that would hinder quick learning are, on a conceptual level, that sovereign debt ratings, their meanings and their underlying procedures are rather opaque. The set of relevant fundamental variables is an open one, and the interpretation of ever evolving political institutions and processes in unprecedented environments are a dime a dozen. This makes an empirical assessment within a finite time frame virtually impossible. How is anyone to dispute, or refute with facts and data, that a country rating of A+ should actually be or have been an AAA-, when no country has ever defaulted within 5 years of being given a rating in the A-segment of the rating scale?

Therefore, the effect that an erroneous or deliberate but unjustified sovereign debt rating downgrade has on a country’s interest rate and financial situation may be permanent or at least highly persistent.

Further estimates, provided in GGJ, illustrate the effect described in Figure 3, and suggest that the situation may even be more serious.

One question that GGJ asked in this respect was whether credit rating agencies applied the rating algorithm identified by equation (6) consistently over time and to all countries, or whether some countries were singled out for special treatment, for better or worse. It turned out that the so-called PIGS countries, i.e. Portugal, Ireland, Greece and Spain, were indeed treated much more harshly than the remaining OECD countries in the sample. This is revealed in the estimation equation

\[
r = 2.41 + 1.00 \text{CRISIS} + 2.30 \text{CRISIS} \times PIGS + 0.38 i_{-1} + bN
\]

(0.73) (0.28) (0.60) (0.08)

\[R^2 = 0.68 \quad n = 262\]  

where CRISIS and PIGS are dummy variables. CRISIS assumes a value of 1 when the country experienced a crisis. These include both successful practitioners and eminent academics. Soros (2012), a legend in investment circles, quips: “I am not well qualified to criticize the theory of rational expectations and the efficient market hypothesis because as a market participant I considered them so unrealistic that I never bothered to study them.” Two of the IMF chief economist Blanchard (2011) four hard truths he learned from 2011 directly bear on this. His number 3 is: “financial investors are schizophrenic about fiscal consolidation and growth”. And the fourth reads: “Perception moulds reality”. See also Arezki, Candelon & Sy (2011).

See Cornaggia, Cornaggia & Hund (2012), Table III.
in the years 2009 and 2010, being zero otherwise. PIGS takes on a value of 1 for Portugal, Ireland, Greece and Spain and is zero for all other countries. The estimated coefficients suggest, first, that during the current sovereign debt crisis rating agencies became more critical in their judgements, grading countries one notch lower for given interest rates and other variables than prior to 2009. In addition, the group of PIGS countries was rated another 2.3 notches lower than other countries. The qualitative result would mimic what we sketched in Figure 4, raising interest rates of PIGS countries in particular, and driving those countries closer towards default.

This effect is aggravated for the PIGS countries because the estimates also suggest that, in the turbulence of this crisis, markets responded to rating downgrades of the PIGS countries much more strongly than when other countries were involved. A downgrade by one notch raised the interest rates of PIGS countries by almost two percentage points over Germany’s:

\[ i - i_D = -8.64 + 1.77 r^{\text{PIGS}} + aM \]

\( R^2 = 0.38 \quad n = 262 \) \hspace{1cm} (8)

While this leaves the qualitative consequences of the shift of the ratings line to the right unchanged, it makes the situation of PIGS countries worse, for two reasons (see Figure 4):

- The negative, self-fulfilling response to a given initial downgrade becomes much stronger. In the case described by equations (5) and (6), an arbitrary downgrade that shifts the \( r \) line to the right by one notch will eventually raise \( r \) by 1.2 and the interest rate by 0.5 percentage points. Using the respective coefficients from equations (8) and (7), the same initial hike of the credit rating by one notch triggers an increase in \( r \) by 3.05, which pushes the interest rate up by a whopping 5.4 percentage points.

- When the \( r \) line shifts to the right without fundamental justification, the absolute difference between the interest rate \( i_2 \) that actually obtains

\(^{18}\)When we add these two effects, the PIGS countries were graded 3.3 notches lower than they would have been before the crisis with the same political and economic data. In regressions where ratings are treated as ordered rather than scaled variables, this unexplained downgrade of Portugal, Ireland, Greece and Spain as a group even widens to 4.84 rating classes.

\(^{19}\)GGJ split the debt rating into three components: A part that can be attributed to fundamentals. A white noise error term. And the part by which the rating of PIGS countries in 2009 and 2010 deviated from the rating algorithm applied previously and to other countries. The first two parts are not shown here.
and the true interest rate \( i'2 \) (that would justify the new rating) remains unchanged (since \( i'2 - i2 = i'1 - i1 \)). But since at \( i2 \) the interest rate is now much higher, the unjustified part is smaller as a percentage of the actual interest rate. In terms of the sovereign debt rating, the country has been downgraded by 3.05 rating categories, of which 2.05 are justified by the triggered increase in the interest rate. The error signal is, therefore, much weaker and the market is therefore even less likely to learn and remedy things than in the numerical example described by equations (5) and (6).

So what these estimates appear to suggest is, in a nutshell:

- downgrades of the main casualties of the eurozone debt crisis cannot be justified on the basis of previous rating algorithms as identified by econometric analysis;

- financial markets in ‘panic mode’ appear to respond much more strongly to signals from rating agencies than they would under normal circumstances;

- the latter result makes it more difficult to detect rating errors or abuses, which would be necessary in order to stop the spiral of self-fulfilling downgrades.

The relevant coefficient estimates provided in GGJ may be questioned on the grounds that only linear relationships are permitted. This leaves room for an alternative explanation of the results. Maybe markets were not really oversensitive to the ratings of the PIGS countries? Instead, the \( i \) line could be nonlinear. It may well be that a change from AAA to AA+ does not call for the same change in the risk premium as a downgrade from, say, BBB- to BB+, or from CCC- to CC. The next section looks at this possibility and explores the implications.

5 An empirical model of interest rate and sovereign bond ratings interaction: Take 2

This section complements and extends the relevant regression results provided in GGJ in three ways:

First, we add observations for 2011. This may not appear all that important from the perspective of overall sample size, which increases by some ten per cent. But it expands the number of observations that we have for the European debt crisis, which only took off in 2009, by one third.
Second, we take a close look at functional forms, permitting nonlinearities in the effects of interest rates on credit ratings, and of credit ratings on interest rates.

Third, we move from a very static view towards a dynamic analysis that permits lagged responses and panic reactions in a market that may not be dominated and driven by full rationality.

5.1 The data

Our empirical analysis uses annual data for 25 OECD countries for the period 1999 to 2011.\textsuperscript{20} Nine OECD members were omitted because no data was available or because they became members after 1999. We chose this specific period because sovereign ratings for the observed countries are not always available before then, and in order to avoid the structural break due to the introduction of the Euro in 11 countries of our sample. The following variables are included, with descriptive statistics shown in Table 1.\textsuperscript{21}

- \textit{Rating}: Three major agencies provide sovereign ratings: Moody’s, Fitch Ratings and Standard & Poor’s. We use the end-of-year, long-term sovereign debt rating of Fitch Ratings.\textsuperscript{22} We convert the ratings into an equidistant numerical scale running from 1 for D to 21 for AAA as in many other studies on rating agencies, such as Afonso, Gomes & Rother (2007).

- \textit{GDP growth}: Data on real GDP growth is from the OECD Economic Outlook No. 90 Annex Table 1.

- \textit{GDP per capita}: Real GDP per capita measured in thousand current international dollars is from the IMF World Economic Outlook database.

- \textit{Budget surplus}: This variable measures general government financial balances as a percentage of nominal GDP and includes one-off factors

\textsuperscript{20}Our sample includes Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, South Korea, Spain, Sweden, Switzerland, United Kingdom, and the USA.

\textsuperscript{21}Our macroeconomic data for 2011 are mostly estimates provided by the OECD Economic Outlook or the IMF World Economic Outlook database.

\textsuperscript{22}Since the data needed for our analysis were not available for all three agencies we settled for Fitch as a representative. Given the high correlation between the ratings of the three big rating agencies, we do not expect this choice to bear on our main results. Note that, for example Gaillard (2011) report pairwise correlation coefficients between the three rating agencies above 0.97 during 2000 until 2010.
such as sales of mobile phone licences. The source is Annex Table 27 of the OECD Economic Outlook No. 90.

- **Primary surplus (adj):** This variable measures the general government underlying primary balance as a percentage of GDP. It equals government surplus less net interest payments and is adjusted for one-off factors. The source is Annex Table 30 of the OECD Economic Outlook No. 90.

- **Debt ratio:** General government gross debt as a percentage of nominal GDP is taken from Annex Table 32 of the OECD Economic Outlook No. 90.

- **Inflation:** Consumer price inflation is taken from Annex Table 18 of the OECD Economic Outlook No. 90.

- **Bond yield:** This is the annual average of monthly 10-year generic government bond yields as provided by Bloomberg.

- **Credit spread:** The credit spread is calculated as the difference between the December value of the monthly 10-year generic government bond yield of a country and that of Germany.

<table>
<thead>
<tr>
<th>Table 1: Descriptive statistics.</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Maximum</th>
<th>Minimum</th>
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<td>61.1</td>
<td>34.4</td>
<td>211.7</td>
<td>13.7</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.3</td>
<td>2.3</td>
<td>1.5</td>
<td>9.8</td>
<td>-1.7</td>
</tr>
<tr>
<td>Bond yield (i)</td>
<td>4.6</td>
<td>4.4</td>
<td>1.7</td>
<td>19.1</td>
<td>1</td>
</tr>
<tr>
<td>Credit spread (i - i_D)</td>
<td>0.8</td>
<td>0.3</td>
<td>2.5</td>
<td>33.1</td>
<td>-3.6</td>
</tr>
</tbody>
</table>

### 5.2 Estimating the rating equation

We start with the rating equation, which attempts to quantify how interest rates on government bonds and other economic and political variables affect the probability of default as measured by the credit rating of the country. Regression results are shown in Table 2.

Column 1 reports the key result, which is in line with the results reported in GGJ. Sovereign bond ratings are found to depend on a vector of macroeconomic indicators typically used in pertinent empirical research.
Table 2: Regressions explaining sovereign debt ratings.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.214</td>
<td>1.214</td>
<td>1.369</td>
<td>4.363***</td>
</tr>
<tr>
<td></td>
<td>(0.966)</td>
<td>(1.600)</td>
<td>(1.087)</td>
<td>(0.739)</td>
</tr>
<tr>
<td>GDP growth</td>
<td>−0.049</td>
<td>−0.049</td>
<td>−0.075</td>
<td>−0.064</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.077)</td>
<td>(0.066)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>GDP per capita</td>
<td>−0.118***</td>
<td>−0.118***</td>
<td>−0.141***</td>
<td>−0.130***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.022)</td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Budget surplus</td>
<td>−0.013</td>
<td>−0.013</td>
<td>0.004</td>
<td>−0.017</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.037)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Debt ratio</td>
<td>0.022***</td>
<td>0.022***</td>
<td>0.024***</td>
<td>0.016***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Primary surplus</td>
<td>−0.141***</td>
<td>−0.141***</td>
<td>−0.165***</td>
<td>−0.123***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.052)</td>
<td>(0.039)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.178**</td>
<td>0.178**</td>
<td>0.211***</td>
<td>0.273***</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.086)</td>
<td>(0.078)</td>
<td>(0.073)</td>
</tr>
<tr>
<td>(i_{-1})</td>
<td>0.693***</td>
<td>0.693***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.182)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\log(i_{-1}))</td>
<td></td>
<td></td>
<td>2.441***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.395)</td>
<td></td>
</tr>
<tr>
<td>(i_{-1}^2)</td>
<td></td>
<td></td>
<td></td>
<td>0.005***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

|                        |         |         |         |         |
| \(R^2\)               | 0.698   | 0.698   | 0.584   | 0.594   |
| adjusted \(R^2\)     | 0.598   | 0.598   | 0.574   | 0.584   |
| \(F\)-statistic       | 62.709  | 55.194  | 56.788  | 59.049  |
| Observations           | 291     | 291     | 291     | 291     |

Notes: OLS regressions. The dependent variable is Rating. \(r\) denotes government bond yields. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D). See Appendix for entire table. Regression (2) displays White’s heteroscedasticity robust standard errors.
These indicators explain 60 percent of the variance of sovereign bond ratings in our panel. All estimated coefficients possess the expected signs, though not all are significantly different from zero. Ratings are found to improve with higher income growth and income levels, or with better overall and primary budget situations. Ratings deteriorate when the debt ratio, inflation or government bond yields go up. Column 2 reports heteroscedasticity robust errors as suggested by White (1980), which leave the significance levels indicated in column 1 very much intact.

A crucial question from the perspective of the model suggested in sections 3 and 4 is whether the regression line in an $r-i$ diagram is linear or not. We did not find any evidence in support of nonlinearity. Tests looking for a breakpoint did not reject the null hypothesis of a constant coefficient, and nonlinear functional relationships did not improve the fit of the regression equation. Columns 3 and 4 are representative of these efforts, showing that convex or concave shapes of the effect of ratings on bond yields lower confidence and fit levels. This suggests that the ratings curve of our model may be considered linear within the range covered by our panel data.

### 5.3 Estimating the interest rate equation

We now turn to the effect of sovereign bond ratings on interest rate spreads. Baseline results are given in Table 3.

#### Table 3: The effect of ratings on sovereign bond yields (I)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2a)</th>
<th>(2b)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.961$***</td>
<td>$-0.197$*</td>
<td>$-22.888$***</td>
<td>$0.685$***</td>
<td>$0.228$***</td>
</tr>
<tr>
<td></td>
<td>$(0.165)$</td>
<td>$(0.103)$</td>
<td>$(0.918)$</td>
<td>$(0.096)$</td>
<td>$(0.068)$</td>
</tr>
<tr>
<td>$r$</td>
<td>$0.657$***</td>
<td>$0.301$***</td>
<td>$3.119$***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(0.045)$</td>
<td>$(0.032)$</td>
<td>$(0.082)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\exp(r)$</td>
<td></td>
<td></td>
<td></td>
<td>$0.000$***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.000)$</td>
<td></td>
</tr>
<tr>
<td>$r^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$0.006$***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$(0.000)$</td>
</tr>
</tbody>
</table>

Notes: OLS regressions. The dependent variable is credit spread $i - i_D$. $r$ denotes sovereign debt rating. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D). See Appendix for entire table. The combined $R^2$ from regressions (2a) and (2b) is 0.805 and the adjusted $R^2$ is 0.805, too.

The regression reported in the first column proposes a simple linear relationship between the interest rate and the sovereign debt rating and serves as a reference point. It suggests that an AAA rated country, which translates
into a numerical value of 1 for the rating variable, may expect an interest rate spread versus Germany of \(-0.961 + 0.657 = -0.304\). Any downgrade by one notch raises this country’s interest rate by 65.7 basis points. This equation explains 42 percent of the variance in the credit spreads in our sample.

As suggested by the nonlinear functional form of the interest rate equation in the Romer model, shown as equation (1) above, we next explore whether the regression coefficient is really constant over the entire range of rating observations. To this end we test the null hypothesis of a constant slope coefficient, as provided in the first column, against the alternative hypothesis of a break in the regression line. Applying the test proposed in Davies (1987), the null hypothesis of no break was rejected, and the break point was found to lie between a BBB+ and a BBB rating. Regression estimates for the resulting two segments are shown as regressions 2a and 2b in Table 3.

The differences between the two segments are striking. The slope coefficients differ by a ratio of ten to one. While, on average, a rating downgrade by one notch raises interest rates by 0.3 percentage points when ratings are in the range between AAA and A-, which comprises seven categories, a downgrade by one step raises the interest rate by 3.12 percent once the rating has fallen into the B segment or below. Both coefficients are highly significant, though, with t-ratios of 9.41 and 38.04, respectively. Fit levels differ substantially, with adjusted coefficients of determination of 0.239 and 0.997, respectively. These refer to quite different sample sizes, however, which comprise 284 observations in the first segment, and only 7 observations in the second. The empirical relevance of permitting nonlinearity in the relationship between interest rate spreads and ratings is underscored by the increase of the adjusted coefficient of determination for the entire sample, which is 0.418 for the linear model and 0.805 for the combined parts of the segmented regression.

Given that the null hypothesis of a constant slope coefficient was rejected, we tried different functional forms to represent the impact of ratings on spreads. Almost any function that permits a convex shape of the \(i\) curve, as proposed in the Romer diagram, generates a significantly improved fit of the estimation equation. For example, using an exponential function, as reported in regression 3 in Table 3, boosts the adjusted coefficient of determination to 0.576. Similar results are found with logarithmic or reciprocal specifications, or when employing polynomials.

After some experimentation, it turns out that a third order polynomial provides the best results from a goodness-of-fit perspective. Matters are sim-

\footnote{For the employed methodology see Davies (1987) and Muggeo (2003). Regressions were run using the R package provided in Muggeo (2008).}
plified by the fact that the first-order and second-order terms are not significant at conventional levels, so that omitting them does not compromise the fit. Thus, we are left with the simple regression reported in the last column of Table 3. This shows that raising ratings to their third power rather than employing them linearly virtually doubles the adjusted coefficient of determination, from 0.418 to 0.798. The implied nonlinearity is substantial: depriving a country of its AAA status, downgrading it by one notch, raises the interest rate by 0.042 percentage points only. Doing the same thing to a BBB+ country raises the interest rate by 1.302 percentage points. But if a country with a CCC+ rating slides down one more step, the interest rate its creditors require increases by a whopping 5.514 percentage points.

5.4 The existence and nature of multiple equilibria and self-fulfilling prophecy

We may now return to our key questions of whether the developments observed during Europe’s debt crisis support the notions of self-fulfilling prophecy and multiple equilibria. We do so on the basis of the model described and discussed in Section 3, which our estimates attempt to quantify. Figure 5 depicts the $r$ and $i$ lines from our estimated model. The $r$ line in Figures 5(a) and 5(b) is derived from regression 1 in Table 2. The $i$ line in Figure 5(a) represents the segmented regression 2a and 2b in Table 3. Figure 5(b) shows the $i$ line representing regression 4 of Table 3 which uses a third-order polynomial term for the sovereign debt rating. In reality, the position of the $r$ line is determined by a set of exogenous variables, which are different for each country and which change over time. To provide for a synthesized general discussion, these exogenous variables are set to their average values in our panel data set in both panels of Figure 5.

Both empirical models feature two points of intersection between the $r$ and $i$ lines, and identify three equilibria. One of these is a degenerated ‘equilibrium’ in which a country is driven into insolvency. This corresponds to the ‘equilibrium’ we labelled Z in Figure 1 above. The first and the second equilibria replicate and have the same properties as equilibrium points X and Y in Figure 1. Point X is a good equilibrium which is locally stable, i.e. stable as long as ratings remain below $r_Y$. Thus, Y marks a vital threshold, a point of no return. Once Y is crossed, the country drifts toward insolvency and can only be rescued by exogenous intervention.
Figure 5: The empirical model with different functional forms.

(a) The empirical model with a segmented interest rate line.

(b) The empirical model with a third-order polynomial interest rate line.
6 Refinements and lessons

We now move beyond the model described in section 3 and look at some lessons suggested by our estimation results.

6.1 Refinements

The estimates provided in regressions 5 - 8 of Table 4 derive from a more flexible interpretation of the Romer model. Regression 5 explores the possibility that interest rates only partially adjust to rating changes by using the lagged credit spread as an additional regressor. The result is a substantial increase of the coefficient of determination from 0.798 to 0.890 due to a highly significant autoregressive coefficient. What this means, essentially, is that the short-run and long-run $i$ curves differ. The equation for the short-run $i$ line is

$$i - i_D = -0.003 + 0.669(i - i_D)_{-1} + 0.004r^3$$ \hspace{1cm} (9)

This line is flatter than the $i$ line in Figure 5(b), where the relevant term is $0.006r^3$ (see regression 4 in Table 3). The long-run relationship, however, is given by

$$i - i_D = -0.009 + 0.012r^3$$ \hspace{1cm} (10)

This equation describes a line that is now twice as steep as the $i$ line depicted in Figure 5(a). One way to interpret this result is the following:

*Stochastic, short-lived errors* in the debt rating of a country have smaller immediate effects on the interest rate than is suggested by the $i$ line in Figure 5(b). This reduces the risk of being pushed beyond the insolvency threshold as a consequence of mistakes by rating agencies alone.

*Persistent rating errors*, or even the strategic use of rating downgrades, may result in substantial increases in the interest rates governments pay on public debt. In such a scenario the insolvency threshold would be much lower than is suggested by Figure 5(b) and the risk of dropping into a default trap would be much higher.

Regressions 6 and 7 explore another possible refinement of our estimates. The hypothesis tested here is whether any rating change that brings a country into the news and unsettles the financial markets has an effect on interest rates. This effect may be independent of the actual debt rating of a country, and would only exist during the period in which the downgrade was announced. Augmenting our equations by an explanatory variable $\Delta r$ does indeed generate a highly significant coefficient and the fit improves substantially. In the case without the lagged endogenous variable (regression 6), the coefficient of determination increases from 0.798 to 0.811. When lagged
Table 4: The effect of ratings on sovereign bond yields (II)

<table>
<thead>
<tr>
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<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.228***</td>
<td>-0.003</td>
<td>0.265***</td>
<td>0.039</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.068)</td>
<td>(0.052)</td>
<td>(0.067)</td>
<td>(0.047)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>$r^3$</td>
<td>0.006***</td>
<td>0.004***</td>
<td>0.005***</td>
<td>0.003***</td>
<td>0.003***</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$(i - i_D)_{-1}$</td>
<td>0.669***</td>
<td>0.704***</td>
<td>0.701***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.039)</td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta r$</td>
<td></td>
<td>0.504***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.114)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta r^+$</td>
<td></td>
<td></td>
<td></td>
<td>0.869***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.094)</td>
<td></td>
</tr>
<tr>
<td>$\Delta r^-$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.152)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.799</td>
<td>0.891</td>
<td>0.813</td>
<td>0.913</td>
<td>0.917</td>
</tr>
<tr>
<td>adjusted $R^2$</td>
<td>0.798</td>
<td>0.890</td>
<td>0.811</td>
<td>0.912</td>
<td>0.916</td>
</tr>
<tr>
<td>$F$-statistic</td>
<td>1146.460</td>
<td>1172.942</td>
<td>622.177</td>
<td>998.264</td>
<td>786.288</td>
</tr>
<tr>
<td>Observations</td>
<td>291</td>
<td>291</td>
<td>290</td>
<td>290</td>
<td>290</td>
</tr>
</tbody>
</table>

Notes: See notes for Table 3. Regression 4 is repeated for convenience. $\Delta r = r - r_{-1}$ denotes rating changes. $\Delta r^+ = \max(0, \Delta r)$ denotes downgrades, whereas $\Delta r^- = \min(0, \Delta r)$ denotes upgrades, only.

When regression equation 8 splits $\Delta r$, which includes rating downgrades as well as upgrades, into $\Delta r^+$ (i.e. $\Delta r > 0$) and $\Delta r^-$ (i.e. $\Delta r < 0$), the coefficient of determination creeps up still further. The estimated coefficients differ in magnitude and are only significant when rating downgrades are made public. The short-run $i$ line given by this regression is

$$i - i_D = 0.002 + 0.701(i - i_D)_{-1} + 0.003r^3 + 0.869\Delta r^+ + 0.172\Delta r^-$$  (11)

This implies a long-run equation that is similar to the one shown above, namely

$$i - i_D = 0.007 + 0.01r^3$$  (12)

The presence in equation (11) of $\Delta r^+$, the coefficient of which carries a $t$-statistic of 9.24, generates some interesting and potentially disquieting dynamics. The immediate response of the interest rate to a rating downgrade is given by

$$\frac{\Delta i}{\Delta r} \approx 0.009r^2 + 0.869$$  (13)

Now recall that the slope of the rating line represented by regression 1 in Table 2 is 1.443. According to equation (13), the interest rate line is steeper than this at levels of $r \geq 8$. This means that at sovereign debt ratings outside spreads are included (regression 7), the goodness-of-fit statistic increases from 0.890 to 0.912.

It may not come as a surprise that these shock effects are not symmetric. When regression equation 8 splits $\Delta r$, which includes rating downgrades as well as upgrades, into $\Delta r^+$ (i.e. $\Delta r > 0$) and $\Delta r^-$ (i.e. $\Delta r < 0$), the coefficient of determination creeps up still further. The estimated coefficients differ in magnitude and are only significant when rating downgrades are made public. The short-run $i$ line given by this regression is

$$i - i_D = 0.002 + 0.701(i - i_D)_{-1} + 0.003r^3 + 0.869\Delta r^+ + 0.172\Delta r^-$$  (11)

This implies a long-run equation that is similar to the one shown above, namely

$$i - i_D = 0.007 + 0.01r^3$$  (12)

The presence in equation (11) of $\Delta r^+$, the coefficient of which carries a $t$-statistic of 9.24, generates some interesting and potentially disquieting dynamics. The immediate response of the interest rate to a rating downgrade is given by

$$\frac{\Delta i}{\Delta r} \approx 0.009r^2 + 0.869$$  (13)

Now recall that the slope of the rating line represented by regression 1 in Table 2 is 1.443. According to equation (13), the interest rate line is steeper than this at levels of $r \geq 8$. This means that at sovereign debt ratings outside

---

24 This is an approximation, of course, since we are dealing in discrete time.
the A-segment, i.e. of BBB+ or worse, a downgrade generates an increase in the interest rate that justifies or more than justifies the initial downgrade, and may trigger a spiral of successive and eventually disastrous downgrades. Only countries in the A-segment of the rating scale appear to be safe from this, at least when the shocks to which they are exposed are only small. However, this only applies when marginal rating shocks occur. Larger shocks, and these have not been the exceptions during Europe’s sovereign debt crisis, may even jeopardize countries which were in secure A territory. We may illustrate this by looking at the impulse responses of equation (11) to shocks of various kinds and magnitudes. This provides us with insolvency thresholds that identify the size of a rating downgrade required to destabilize the public finances of countries with a given sovereign debt rating. Figure 6 reports the results for the case of rating shocks.

Figure 6(a) looks at temporary rating shocks, as they would be captured by the error term of our regression equation. The vertical line from a BBB rating upwards indicates that the equilibrium is inherently unstable and that the smallest of shocks suffices to trigger an accelerating debt crisis. The outward-sloping segment shows how rating shocks to a country located in the range A- and better need to be increasingly larger to destabilize the country. For example, a country with a AA (= 3) rating would have to be subjected to an arbitrary downgrade of six notches to a BB- rating in order to be pushed beyond the insolvency threshold.

When rating shocks last, however, as has apparently been the case for the eurozone’s PIGS members, much smaller unsubstantiated rating changes may play havoc with government bond markets and suffice to run initially healthy countries into trouble, as shown in Figure 6(b). In this scenario, an arbitrary, yet persistent, downgrade by two notches would trigger a downward spiral in a country with an AA rating. Rising interest rates would call for further downgrades, which would appear to justify the initial downgrade as an apparently good forecast.

The thresholds depicted in Figure 6 are conservative in the sense that they overestimate the shocks needed to destabilize countries. This is because we were focusing on the interaction between ratings and interest rates alone. All other variables that affect sovereign debt ratings were considered exogenous and thus kept constant during our simulations. In reality, the interest rate hikes that follow rating downgrades will increase budget deficits and debt ratios, and may depress economic activity in general. All this has an additional negative effect on a country’s rating, and will thus reinforce the negative tendencies in the country’s public finances. With this added transmission channel, even smaller shocks may already suffice to trigger sovereign debt crises.
Figure 6: Insolvency thresholds under temporary and permanent rating shocks.
6.2 Lessons

To the extent that Figures 5(a) and 5(b) provide useful descriptions and simplifications of the structure, equilibria, dynamics and comparative statics underlying the eurozone’s sovereign debt crisis, such crises may stem from two sources:

1. The estimated relationships are stochastic. Any temporary or permanent deviations or changes, as reflected in the error or constant terms of the estimation equations, may drive the country out of an initially stable neighbourhood and trigger a crisis. Section 6.1 looked at rating shocks and the damage they might do.

2. Any changes in the ‘exogenous’ variables that affect the positions of the i line or the r line and, thus, the equilibria, may make a country more vulnerable to a sovereign debt crisis. For instance, any change that increases the risk-free rate, moving the i line up, or changes that shift the rating line to the right, have two unfavourable effects. First, they render the good equilibrium less ‘good’, raising the associated interest rate and credit rating. Second, they move the insolvency threshold to the left, increasing the risk of being pushed into bankruptcy by unfavourable developments. If the relative effects are strong enough, the two curves may lose contact, making the good equilibrium and the threshold disappear, and rendering bankruptcy unavoidable.

Section 6.1 looked at rating shocks and suggested that they may have played a major role in the gestation and propulsion of Europe’s sovereign debt crisis. A rating shock of 3.3 notches, to which the PIGS countries were subjected at the start of the crisis, according to GGJ, would constitute a serious threat to all but the most highly rated countries. At the beginning and during the crisis, however, these risks were aggravated by budgetary and income shocks instigated by the housing and financial crisis. Coefficients from regression 1 in Table 2 permit a first quantitative assessment of the impact of these shocks on individual countries. A look at individual PIGS countries reveals the following. Deteriorating fundamentals shifted Portugal’s rating curve to the right by 0.62 rating notches between 2009 and 2011. However, the country was downgraded by 8 notches during that time. For Ireland the line shift was 1.32 during those years, but the rating dropped by 7 notches. Greece’s rating curve shifted by 0.15 notches, whereas the country was dealt a hefty downgrade of 12 notches. And Spain, finally, was downgraded by three notches from AA+ to AA-, while its rating line only shifted by 0.46 units to the right. In the context of the results reported in Figure 6, this
suggests that budgetary and income shocks may have played a minor role only, and that exceptional changes in the risk assessment of the markets and rating agencies were a key factor in Europe’s debt crisis.

7 Summary and conclusions

This paper analysed the European sovereign debt crisis that grew out of the global real estate and financial crisis of 2007-2009. Drawing on data for 25 OECD countries for the period between 1999 and 2011, we specifically asked whether there was evidence of multiple equilibria and self-fulfilling prophecy in the market for sovereign bonds. Special attention was given to the role of rating agencies and to nonlinearities and dynamics in the interaction between government bond yields and sovereign bond ratings.

We find robust evidence of a nonlinear effect of ratings on interest rates that reflects the theory. This nonlinearity is strong enough to generate multiple equilibria. This, in turn, may render rating errors or abuses, or market panic stemming from other sources, self-fulfilling in a strict sense. In the implied good and stable equilibrium, ratings are excellent and interest rates are low. A second equilibrium looms, which is unstable. It constitutes a threshold beyond which the country falls into an insolvency trap from which it may only escape by exogenous policy measures or outside intervention.

A more detailed look at the dynamics of the effect of debt rating downgrades on interest rates revealed that at least for countries with sovereign debt ratings outside the A range even erroneous, arbitrary or abusive rating downgrades may easily generate the very conditions that do actually justify the rating. Combined with earlier evidence that many of the rating downgrades of the eurozone’s peripheral countries appeared arbitrary and could not be justified on the basis of rating algorithms that explain the ratings of other countries or ratings before 2009, this result is highly discomforting. It urges governments to take a long overdue close look at financial markets in general, and at sovereign bond markets in particular, and at the motivations, dependencies and conflicts of interest of key players in these markets.
### Appendix

#### A.1 Rating conversion

Table 5: Rating conversion.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Numerical value</th>
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<tr>
<td>AAA</td>
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</tr>
<tr>
<td>AA+</td>
<td>2</td>
</tr>
<tr>
<td>AA</td>
<td>3</td>
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<td>AA-</td>
<td>4</td>
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<tr>
<td>A+</td>
<td>5</td>
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<td>A</td>
<td>6</td>
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<tr>
<td>A-</td>
<td>7</td>
</tr>
<tr>
<td>BBB+</td>
<td>8</td>
</tr>
<tr>
<td>BBB</td>
<td>9</td>
</tr>
<tr>
<td>BBB-</td>
<td>10</td>
</tr>
<tr>
<td>BB+</td>
<td>11</td>
</tr>
<tr>
<td>BB</td>
<td>12</td>
</tr>
<tr>
<td>BB-</td>
<td>13</td>
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<tr>
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<td>B</td>
<td>15</td>
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<td>D</td>
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